VariableSelection

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# Variable Selection is an important inferential technique to identify the important variables for a regression. In the traditional set up we often make use of the Stepwise Forward and Backword procedures.

#Stepwise Forward Regression We develop regression model from a set of candidate covariates by including them in the model according to the corresponding p-values in a stepwise manner. We stop the process of including more variable until there is no variable left to enter any more based on certain criteria. If details is set to TRUE in the R function , each step is displayed. We make use of the R package library(olsrr)

## Loading the Package and the Dataset

library(olsrr)

##   
## Attaching package: 'olsrr'

## The following object is masked from 'package:datasets':  
##   
## rivers

data("surgical")  
summary(surgical)

## bcs pindex enzyme\_test liver\_test   
## Min. : 2.600 Min. : 8.00 Min. : 23.00 Min. :0.740   
## 1st Qu.: 5.025 1st Qu.:52.50 1st Qu.: 67.25 1st Qu.:2.020   
## Median : 5.800 Median :63.00 Median : 79.00 Median :2.595   
## Mean : 5.783 Mean :63.24 Mean : 77.11 Mean :2.744   
## 3rd Qu.: 6.500 3rd Qu.:76.00 3rd Qu.: 89.50 3rd Qu.:3.275   
## Max. :11.200 Max. :96.00 Max. :119.00 Max. :6.400   
## age gender alc\_mod alc\_heavy   
## Min. :30.00 Min. :0.000 Min. :0.000 Min. :0.0000   
## 1st Qu.:44.25 1st Qu.:0.000 1st Qu.:0.000 1st Qu.:0.0000   
## Median :51.50 Median :0.000 Median :1.000 Median :0.0000   
## Mean :51.61 Mean :0.463 Mean :0.537 Mean :0.1852   
## 3rd Qu.:60.50 3rd Qu.:1.000 3rd Qu.:1.000 3rd Qu.:0.0000   
## Max. :70.00 Max. :1.000 Max. :1.000 Max. :1.0000   
## y   
## Min. : 181.0   
## 1st Qu.: 482.0   
## Median : 605.5   
## Mean : 702.1   
## 3rd Qu.: 750.5   
## Max. :2343.0

## Conductiing the Stepwise forward procedure based on p-values

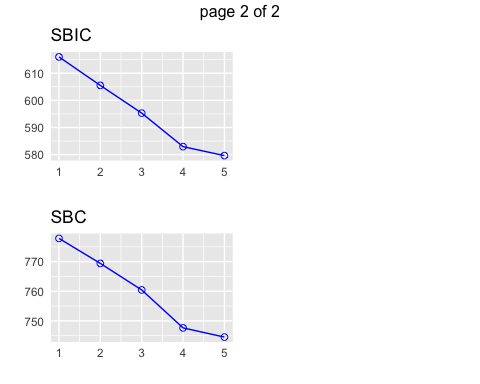
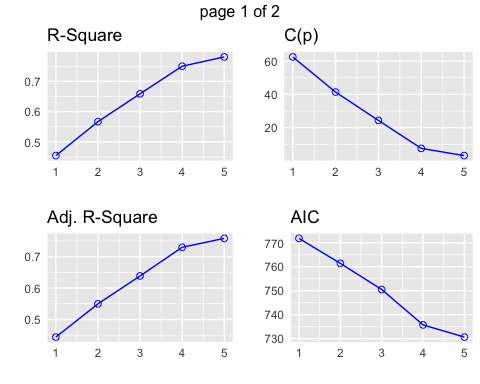
## Stepwise forward model  
model <- lm(y ~ ., data = surgical, details=FALSE)

## Warning: In lm.fit(x, y, offset = offset, singular.ok = singular.ok, ...) :  
## extra argument 'details' will be disregarded

step\_for\_pvalue<-ols\_step\_forward\_p(model)  
step\_for\_pvalue

##   
## Selection Summary   
## ------------------------------------------------------------------------------  
## Variable Adj.   
## Step Entered R-Square R-Square C(p) AIC RMSE   
## ------------------------------------------------------------------------------  
## 1 liver\_test 0.4545 0.4440 62.5119 771.8753 296.2992   
## 2 alc\_heavy 0.5667 0.5498 41.3681 761.4394 266.6484   
## 3 enzyme\_test 0.6590 0.6385 24.3379 750.5089 238.9145   
## 4 pindex 0.7501 0.7297 7.5373 735.7146 206.5835   
## 5 bcs 0.7809 0.7581 3.1925 730.6204 195.4544   
## ------------------------------------------------------------------------------

plot(step\_for\_pvalue)



## Conductiing the Stepwise forward procedure based on AIC

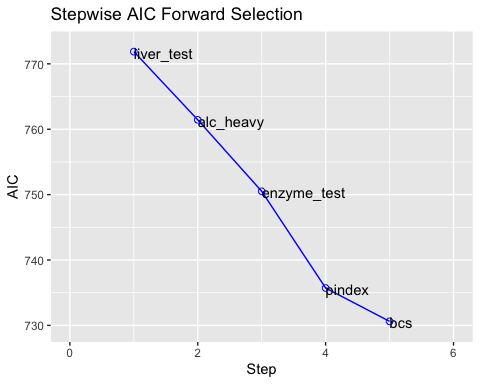
## Stepwise forward model  
model <- lm(y ~ ., data = surgical, details=FALSE)

## Warning: In lm.fit(x, y, offset = offset, singular.ok = singular.ok, ...) :  
## extra argument 'details' will be disregarded

step\_for\_aic<-ols\_step\_forward\_aic(model)  
step\_for\_aic

##   
## Selection Summary   
## ----------------------------------------------------------------------------  
## Variable AIC Sum Sq RSS R-Sq Adj. R-Sq   
## ----------------------------------------------------------------------------  
## liver\_test 771.875 3804272.477 4565248.060 0.45454 0.44405   
## alc\_heavy 761.439 4743349.776 3626170.761 0.56674 0.54975   
## enzyme\_test 750.509 5515514.136 2854006.401 0.65900 0.63854   
## pindex 735.715 6278360.060 2091160.477 0.75015 0.72975   
## bcs 730.620 6535804.090 1833716.447 0.78091 0.75808   
## ----------------------------------------------------------------------------

plot(step\_for\_aic)



## Conductiing the Stepwise forward procedure based on AIC (and similarly based on p-values)

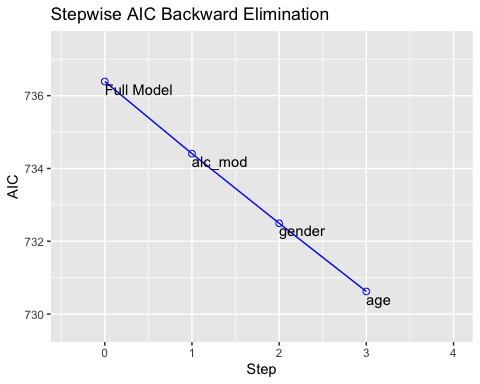
## Stepwise forward model  
model <- lm(y ~ ., data = surgical, details=FALSE)

## Warning: In lm.fit(x, y, offset = offset, singular.ok = singular.ok, ...) :  
## extra argument 'details' will be disregarded

step\_back\_aic<-ols\_step\_backward\_aic(model)  
step\_back\_aic

##   
##   
## Backward Elimination Summary   
## ---------------------------------------------------------------------------  
## Variable AIC RSS Sum Sq R-Sq Adj. R-Sq   
## ---------------------------------------------------------------------------  
## Full Model 736.390 1825905.713 6543614.824 0.78184 0.74305   
## alc\_mod 734.407 1826477.828 6543042.709 0.78177 0.74856   
## gender 732.494 1829435.617 6540084.920 0.78142 0.75351   
## age 730.620 1833716.447 6535804.090 0.78091 0.75808   
## ---------------------------------------------------------------------------

plot(step\_back\_aic)



# For detailsed out put of the procedure:

## Conductiing the Stepwise forward procedure based on AIC

## Stepwise forward model  
model <- lm(y ~ ., data = surgical)  
step\_for\_aic<-ols\_step\_forward\_aic(model,details=TRUE)

## Forward Selection Method   
## ------------------------  
##   
## Candidate Terms:   
##   
## 1 . bcs   
## 2 . pindex   
## 3 . enzyme\_test   
## 4 . liver\_test   
## 5 . age   
## 6 . gender   
## 7 . alc\_mod   
## 8 . alc\_heavy   
##   
## Step 0: AIC = 802.606   
## y ~ 1   
##   
## --------------------------------------------------------------------------------  
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq   
## --------------------------------------------------------------------------------  
## liver\_test 1 771.875 3804272.477 4565248.060 0.455 0.444   
## enzyme\_test 1 782.629 2798309.881 5571210.656 0.334 0.322   
## pindex 1 794.100 1479766.754 6889753.784 0.177 0.161   
## alc\_heavy 1 794.301 1454057.255 6915463.282 0.174 0.158   
## bcs 1 797.697 1005151.658 7364368.879 0.120 0.103   
## alc\_mod 1 802.828 271062.330 8098458.207 0.032 0.014   
## gender 1 802.956 251808.570 8117711.967 0.030 0.011   
## age 1 803.834 118862.559 8250657.978 0.014 -0.005   
## --------------------------------------------------------------------------------  
##   
##   
## - liver\_test   
##   
##   
## Step 1 : AIC = 771.8753   
## y ~ liver\_test   
##   
## -------------------------------------------------------------------------------  
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq   
## -------------------------------------------------------------------------------  
## alc\_heavy 1 761.439 939077.300 3626170.761 0.567 0.550   
## enzyme\_test 1 762.077 896004.331 3669243.729 0.562 0.544   
## pindex 1 770.387 285591.786 4279656.274 0.489 0.469   
## alc\_mod 1 771.141 225396.238 4339851.822 0.481 0.461   
## gender 1 773.802 6162.222 4559085.838 0.455 0.434   
## age 1 773.831 3726.297 4561521.763 0.455 0.434   
## bcs 1 773.867 685.256 4564562.805 0.455 0.433   
## -------------------------------------------------------------------------------  
##   
## - alc\_heavy   
##   
##   
## Step 2 : AIC = 761.4394   
## y ~ liver\_test + alc\_heavy   
##   
## -------------------------------------------------------------------------------  
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq   
## -------------------------------------------------------------------------------  
## enzyme\_test 1 750.509 772164.360 2854006.401 0.659 0.639   
## pindex 1 756.125 459358.635 3166812.126 0.622 0.599   
## bcs 1 763.063 25195.587 3600975.173 0.570 0.544   
## age 1 763.110 22048.109 3604122.652 0.569 0.544   
## alc\_mod 1 763.428 784.551 3625386.210 0.567 0.541   
## gender 1 763.433 443.343 3625727.417 0.567 0.541   
## -------------------------------------------------------------------------------  
##   
## - enzyme\_test   
##   
##   
## Step 3 : AIC = 750.5089   
## y ~ liver\_test + alc\_heavy + enzyme\_test   
##   
## -----------------------------------------------------------------------------  
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq   
## -----------------------------------------------------------------------------  
## pindex 1 735.715 762845.924 2091160.477 0.750 0.730   
## bcs 1 750.782 89836.308 2764170.093 0.670 0.643   
## alc\_mod 1 752.403 5607.570 2848398.831 0.660 0.632   
## age 1 752.416 4896.081 2849110.320 0.660 0.632   
## gender 1 752.509 5.958 2854000.443 0.659 0.631   
## -----------------------------------------------------------------------------  
##   
## - pindex   
##   
##   
## Step 4 : AIC = 735.7146   
## y ~ liver\_test + alc\_heavy + enzyme\_test + pindex   
##   
## -----------------------------------------------------------------------------  
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq   
## -----------------------------------------------------------------------------  
## bcs 1 730.620 257444.030 1833716.447 0.781 0.758   
## age 1 737.680 1325.880 2089834.596 0.750 0.724   
## gender 1 737.712 90.186 2091070.290 0.750 0.724   
## alc\_mod 1 737.713 60.620 2091099.857 0.750 0.724   
## -----------------------------------------------------------------------------  
##   
## - bcs   
##   
##   
## Step 5 : AIC = 730.6204   
## y ~ liver\_test + alc\_heavy + enzyme\_test + pindex + bcs   
##   
## ---------------------------------------------------------------------------  
## Variable DF AIC Sum Sq RSS R-Sq Adj. R-Sq   
## ---------------------------------------------------------------------------  
## age 1 732.494 4280.830 1829435.617 0.781 0.754   
## gender 1 732.551 2360.288 1831356.159 0.781 0.753   
## alc\_mod 1 732.614 216.992 1833499.455 0.781 0.753   
## ---------------------------------------------------------------------------  
##   
##   
## No more variables to be added.  
##   
## Variables Entered:   
##   
## - liver\_test   
## - alc\_heavy   
## - enzyme\_test   
## - pindex   
## - bcs   
##   
##   
## Final Model Output   
## ------------------  
##   
## Model Summary   
## -----------------------------------------------------------------  
## R 0.884 RMSE 195.454   
## R-Squared 0.781 Coef. Var 27.839   
## Adj. R-Squared 0.758 MSE 38202.426   
## Pred R-Squared 0.700 MAE 137.656   
## -----------------------------------------------------------------  
## RMSE: Root Mean Square Error   
## MSE: Mean Square Error   
## MAE: Mean Absolute Error   
##   
## ANOVA   
## -----------------------------------------------------------------------  
## Sum of   
## Squares DF Mean Square F Sig.   
## -----------------------------------------------------------------------  
## Regression 6535804.090 5 1307160.818 34.217 0.0000   
## Residual 1833716.447 48 38202.426   
## Total 8369520.537 53   
## -----------------------------------------------------------------------  
##   
## Parameter Estimates   
## ------------------------------------------------------------------------------------------------  
## model Beta Std. Error Std. Beta t Sig lower upper   
## ------------------------------------------------------------------------------------------------  
## (Intercept) -1178.330 208.682 -5.647 0.000 -1597.914 -758.746   
## liver\_test 58.064 40.144 0.156 1.446 0.155 -22.652 138.779   
## alc\_heavy 317.848 71.634 0.314 4.437 0.000 173.818 461.878   
## enzyme\_test 9.748 1.656 0.521 5.887 0.000 6.419 13.077   
## pindex 8.924 1.808 0.380 4.935 0.000 5.288 12.559   
## bcs 59.864 23.060 0.241 2.596 0.012 13.498 106.230   
## ------------------------------------------------------------------------------------------------

step\_for\_aic

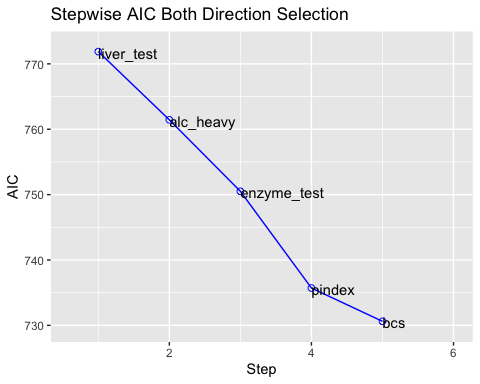
##   
## Selection Summary   
## ----------------------------------------------------------------------------  
## Variable AIC Sum Sq RSS R-Sq Adj. R-Sq   
## ----------------------------------------------------------------------------  
## liver\_test 771.875 3804272.477 4565248.060 0.45454 0.44405   
## alc\_heavy 761.439 4743349.776 3626170.761 0.56674 0.54975   
## enzyme\_test 750.509 5515514.136 2854006.401 0.65900 0.63854   
## pindex 735.715 6278360.060 2091160.477 0.75015 0.72975   
## bcs 730.620 6535804.090 1833716.447 0.78091 0.75808   
## ----------------------------------------------------------------------------

## Conductiing the Stepwise forward and backward in a optimal combination procedure based on AIC

## Stepwise forward model  
model <- lm(y ~ ., data = surgical)  
step\_for\_aic<-ols\_step\_both\_aic(model, details=FALSE)  
step\_for\_aic

##   
##   
## Stepwise Summary   
## ----------------------------------------------------------------------------------------  
## Variable Method AIC RSS Sum Sq R-Sq Adj. R-Sq   
## ----------------------------------------------------------------------------------------  
## liver\_test addition 771.875 4565248.060 3804272.477 0.45454 0.44405   
## alc\_heavy addition 761.439 3626170.761 4743349.776 0.56674 0.54975   
## enzyme\_test addition 750.509 2854006.401 5515514.136 0.65900 0.63854   
## pindex addition 735.715 2091160.477 6278360.060 0.75015 0.72975   
## bcs addition 730.620 1833716.447 6535804.090 0.78091 0.75808   
## ----------------------------------------------------------------------------------------

plot(step\_for\_aic)



# For consider all the models and identifying the best

model <- lm(y ~ ., data = surgical)  
# This takes time as it will consider all 2^p-1 models  
all\_models <- ols\_step\_all\_possible(model)  
all\_models

## Index N Predictors  
## 4 1 1 liver\_test  
## 3 2 1 enzyme\_test  
## 2 3 1 pindex  
## 8 4 1 alc\_heavy  
## 1 5 1 bcs  
## 7 6 1 alc\_mod  
## 6 7 1 gender  
## 5 8 1 age  
## 30 9 2 liver\_test alc\_heavy  
## 22 10 2 enzyme\_test liver\_test  
## 10 11 2 bcs enzyme\_test  
## 16 12 2 pindex enzyme\_test  
## 17 13 2 pindex liver\_test  
## 29 14 2 liver\_test alc\_mod  
## 26 15 2 enzyme\_test alc\_heavy  
## 28 16 2 liver\_test gender  
## 27 17 2 liver\_test age  
## 11 18 2 bcs liver\_test  
## 21 19 2 pindex alc\_heavy  
## 25 20 2 enzyme\_test alc\_mod  
## 23 21 2 enzyme\_test age  
## 24 22 2 enzyme\_test gender  
## 9 23 2 bcs pindex  
## 15 24 2 bcs alc\_heavy  
## 20 25 2 pindex alc\_mod  
## 35 26 2 gender alc\_heavy  
## 19 27 2 pindex gender  
## 18 28 2 pindex age  
## 33 29 2 age alc\_heavy  
## 36 30 2 alc\_mod alc\_heavy  
## 13 31 2 bcs gender  
## 14 32 2 bcs alc\_mod  
## 12 33 2 bcs age  
## 34 34 2 gender alc\_mod  
## 31 35 2 age gender  
## 32 36 2 age alc\_mod  
## 37 37 3 bcs pindex enzyme\_test  
## 62 38 3 pindex enzyme\_test alc\_heavy  
## 76 39 3 enzyme\_test liver\_test alc\_heavy  
## 58 40 3 pindex enzyme\_test liver\_test  
## 66 41 3 pindex liver\_test alc\_heavy  
## 43 42 3 bcs enzyme\_test liver\_test  
## 47 43 3 bcs enzyme\_test alc\_heavy  
## 75 44 3 enzyme\_test liver\_test alc\_mod  
## 51 45 3 bcs liver\_test alc\_heavy  
## 85 46 3 liver\_test age alc\_heavy  
## 88 47 3 liver\_test alc\_mod alc\_heavy  
## 87 48 3 liver\_test gender alc\_heavy  
## 74 49 3 enzyme\_test liver\_test gender  
## 73 50 3 enzyme\_test liver\_test age  
## 61 51 3 pindex enzyme\_test alc\_mod  
## 44 52 3 bcs enzyme\_test age  
## 46 53 3 bcs enzyme\_test alc\_mod  
## 59 54 3 pindex enzyme\_test age  
## 45 55 3 bcs enzyme\_test gender  
## 65 56 3 pindex liver\_test alc\_mod  
## 60 57 3 pindex enzyme\_test gender  
## 38 58 3 bcs pindex liver\_test  
## 64 59 3 pindex liver\_test gender  
## 63 60 3 pindex liver\_test age  
## 84 61 3 liver\_test age alc\_mod  
## 86 62 3 liver\_test gender alc\_mod  
## 50 63 3 bcs liver\_test alc\_mod  
## 81 64 3 enzyme\_test gender alc\_heavy  
## 79 65 3 enzyme\_test age alc\_heavy  
## 82 66 3 enzyme\_test alc\_mod alc\_heavy  
## 83 67 3 liver\_test age gender  
## 49 68 3 bcs liver\_test gender  
## 48 69 3 bcs liver\_test age  
## 42 70 3 bcs pindex alc\_heavy  
## 71 71 3 pindex gender alc\_heavy  
## 69 72 3 pindex age alc\_heavy  
## 72 73 3 pindex alc\_mod alc\_heavy  
## 80 74 3 enzyme\_test gender alc\_mod  
## 78 75 3 enzyme\_test age alc\_mod  
## 77 76 3 enzyme\_test age gender  
## 41 77 3 bcs pindex alc\_mod  
## 40 78 3 bcs pindex gender  
## 39 79 3 bcs pindex age  
## 56 80 3 bcs gender alc\_heavy  
## 70 81 3 pindex gender alc\_mod  
## 54 82 3 bcs age alc\_heavy  
## 57 83 3 bcs alc\_mod alc\_heavy  
## 68 84 3 pindex age alc\_mod  
## 90 85 3 age gender alc\_heavy  
## 92 86 3 gender alc\_mod alc\_heavy  
## 67 87 3 pindex age gender  
## 91 88 3 age alc\_mod alc\_heavy  
## 55 89 3 bcs gender alc\_mod  
## 52 90 3 bcs age gender  
## 53 91 3 bcs age alc\_mod  
## 89 92 3 age gender alc\_mod  
## 97 93 4 bcs pindex enzyme\_test alc\_heavy  
## 131 94 4 pindex enzyme\_test liver\_test alc\_heavy  
## 96 95 4 bcs pindex enzyme\_test alc\_mod  
## 94 96 4 bcs pindex enzyme\_test age  
## 93 97 4 bcs pindex enzyme\_test liver\_test  
## 95 98 4 bcs pindex enzyme\_test gender  
## 136 99 4 pindex enzyme\_test gender alc\_heavy  
## 134 100 4 pindex enzyme\_test age alc\_heavy  
## 137 101 4 pindex enzyme\_test alc\_mod alc\_heavy  
## 111 102 4 bcs enzyme\_test liver\_test alc\_heavy  
## 153 103 4 enzyme\_test liver\_test alc\_mod alc\_heavy  
## 150 104 4 enzyme\_test liver\_test age alc\_heavy  
## 152 105 4 enzyme\_test liver\_test gender alc\_heavy  
## 130 106 4 pindex enzyme\_test liver\_test alc\_mod  
## 129 107 4 pindex enzyme\_test liver\_test gender  
## 128 108 4 pindex enzyme\_test liver\_test age  
## 140 109 4 pindex liver\_test age alc\_heavy  
## 101 110 4 bcs pindex liver\_test alc\_heavy  
## 143 111 4 pindex liver\_test alc\_mod alc\_heavy  
## 142 112 4 pindex liver\_test gender alc\_heavy  
## 110 113 4 bcs enzyme\_test liver\_test alc\_mod  
## 116 114 4 bcs enzyme\_test gender alc\_heavy  
## 114 115 4 bcs enzyme\_test age alc\_heavy  
## 108 116 4 bcs enzyme\_test liver\_test age  
## 109 117 4 bcs enzyme\_test liver\_test gender  
## 117 118 4 bcs enzyme\_test alc\_mod alc\_heavy  
## 151 119 4 enzyme\_test liver\_test gender alc\_mod  
## 149 120 4 enzyme\_test liver\_test age alc\_mod  
## 120 121 4 bcs liver\_test age alc\_heavy  
## 123 122 4 bcs liver\_test alc\_mod alc\_heavy  
## 122 123 4 bcs liver\_test gender alc\_heavy  
## 159 124 4 liver\_test age gender alc\_heavy  
## 160 125 4 liver\_test age alc\_mod alc\_heavy  
## 161 126 4 liver\_test gender alc\_mod alc\_heavy  
## 148 127 4 enzyme\_test liver\_test age gender  
## 133 128 4 pindex enzyme\_test age alc\_mod  
## 135 129 4 pindex enzyme\_test gender alc\_mod  
## 113 130 4 bcs enzyme\_test age alc\_mod  
## 112 131 4 bcs enzyme\_test age gender  
## 115 132 4 bcs enzyme\_test gender alc\_mod  
## 132 133 4 pindex enzyme\_test age gender  
## 139 134 4 pindex liver\_test age alc\_mod  
## 141 135 4 pindex liver\_test gender alc\_mod  
## 100 136 4 bcs pindex liver\_test alc\_mod  
## 99 137 4 bcs pindex liver\_test gender  
## 138 138 4 pindex liver\_test age gender  
## 98 139 4 bcs pindex liver\_test age  
## 158 140 4 liver\_test age gender alc\_mod  
## 119 141 4 bcs liver\_test age alc\_mod  
## 121 142 4 bcs liver\_test gender alc\_mod  
## 155 143 4 enzyme\_test age gender alc\_heavy  
## 157 144 4 enzyme\_test gender alc\_mod alc\_heavy  
## 156 145 4 enzyme\_test age alc\_mod alc\_heavy  
## 118 146 4 bcs liver\_test age gender  
## 106 147 4 bcs pindex gender alc\_heavy  
## 104 148 4 bcs pindex age alc\_heavy  
## 107 149 4 bcs pindex alc\_mod alc\_heavy  
## 145 150 4 pindex age gender alc\_heavy  
## 147 151 4 pindex gender alc\_mod alc\_heavy  
## 146 152 4 pindex age alc\_mod alc\_heavy  
## 154 153 4 enzyme\_test age gender alc\_mod  
## 105 154 4 bcs pindex gender alc\_mod  
## 103 155 4 bcs pindex age alc\_mod  
## 102 156 4 bcs pindex age gender  
## 125 157 4 bcs age gender alc\_heavy  
## 127 158 4 bcs gender alc\_mod alc\_heavy  
## 144 159 4 pindex age gender alc\_mod  
## 126 160 4 bcs age alc\_mod alc\_heavy  
## 162 161 4 age gender alc\_mod alc\_heavy  
## 124 162 4 bcs age gender alc\_mod  
## 166 163 5 bcs pindex enzyme\_test liver\_test alc\_heavy  
## 169 164 5 bcs pindex enzyme\_test age alc\_heavy  
## 171 165 5 bcs pindex enzyme\_test gender alc\_heavy  
## 172 166 5 bcs pindex enzyme\_test alc\_mod alc\_heavy  
## 200 167 5 pindex enzyme\_test liver\_test age alc\_heavy  
## 202 168 5 pindex enzyme\_test liver\_test gender alc\_heavy  
## 203 169 5 pindex enzyme\_test liver\_test alc\_mod alc\_heavy  
## 165 170 5 bcs pindex enzyme\_test liver\_test alc\_mod  
## 168 171 5 bcs pindex enzyme\_test age alc\_mod  
## 170 172 5 bcs pindex enzyme\_test gender alc\_mod  
## 163 173 5 bcs pindex enzyme\_test liver\_test age  
## 167 174 5 bcs pindex enzyme\_test age gender  
## 164 175 5 bcs pindex enzyme\_test liver\_test gender  
## 205 176 5 pindex enzyme\_test age gender alc\_heavy  
## 207 177 5 pindex enzyme\_test gender alc\_mod alc\_heavy  
## 206 178 5 pindex enzyme\_test age alc\_mod alc\_heavy  
## 188 179 5 bcs enzyme\_test liver\_test alc\_mod alc\_heavy  
## 187 180 5 bcs enzyme\_test liver\_test gender alc\_heavy  
## 185 181 5 bcs enzyme\_test liver\_test age alc\_heavy  
## 215 182 5 enzyme\_test liver\_test age alc\_mod alc\_heavy  
## 216 183 5 enzyme\_test liver\_test gender alc\_mod alc\_heavy  
## 201 184 5 pindex enzyme\_test liver\_test gender alc\_mod  
## 214 185 5 enzyme\_test liver\_test age gender alc\_heavy  
## 199 186 5 pindex enzyme\_test liver\_test age alc\_mod  
## 198 187 5 pindex enzyme\_test liver\_test age gender  
## 175 188 5 bcs pindex liver\_test age alc\_heavy  
## 210 189 5 pindex liver\_test age alc\_mod alc\_heavy  
## 209 190 5 pindex liver\_test age gender alc\_heavy  
## 178 191 5 bcs pindex liver\_test alc\_mod alc\_heavy  
## 177 192 5 bcs pindex liver\_test gender alc\_heavy  
## 211 193 5 pindex liver\_test gender alc\_mod alc\_heavy  
## 184 194 5 bcs enzyme\_test liver\_test age alc\_mod  
## 186 195 5 bcs enzyme\_test liver\_test gender alc\_mod  
## 190 196 5 bcs enzyme\_test age gender alc\_heavy  
## 192 197 5 bcs enzyme\_test gender alc\_mod alc\_heavy  
## 191 198 5 bcs enzyme\_test age alc\_mod alc\_heavy  
## 183 199 5 bcs enzyme\_test liver\_test age gender  
## 213 200 5 enzyme\_test liver\_test age gender alc\_mod  
## 194 201 5 bcs liver\_test age gender alc\_heavy  
## 195 202 5 bcs liver\_test age alc\_mod alc\_heavy  
## 196 203 5 bcs liver\_test gender alc\_mod alc\_heavy  
## 218 204 5 liver\_test age gender alc\_mod alc\_heavy  
## 204 205 5 pindex enzyme\_test age gender alc\_mod  
## 189 206 5 bcs enzyme\_test age gender alc\_mod  
## 208 207 5 pindex liver\_test age gender alc\_mod  
## 174 208 5 bcs pindex liver\_test age alc\_mod  
## 176 209 5 bcs pindex liver\_test gender alc\_mod  
## 173 210 5 bcs pindex liver\_test age gender  
## 193 211 5 bcs liver\_test age gender alc\_mod  
## 217 212 5 enzyme\_test age gender alc\_mod alc\_heavy  
## 180 213 5 bcs pindex age gender alc\_heavy  
## 182 214 5 bcs pindex gender alc\_mod alc\_heavy  
## 181 215 5 bcs pindex age alc\_mod alc\_heavy  
## 212 216 5 pindex age gender alc\_mod alc\_heavy  
## 179 217 5 bcs pindex age gender alc\_mod  
## 197 218 5 bcs age gender alc\_mod alc\_heavy  
## 221 219 6 bcs pindex enzyme\_test liver\_test age alc\_heavy  
## 223 220 6 bcs pindex enzyme\_test liver\_test gender alc\_heavy  
## 224 221 6 bcs pindex enzyme\_test liver\_test alc\_mod alc\_heavy  
## 226 222 6 bcs pindex enzyme\_test age gender alc\_heavy  
## 227 223 6 bcs pindex enzyme\_test age alc\_mod alc\_heavy  
## 228 224 6 bcs pindex enzyme\_test gender alc\_mod alc\_heavy  
## 241 225 6 pindex enzyme\_test liver\_test age gender alc\_heavy  
## 242 226 6 pindex enzyme\_test liver\_test age alc\_mod alc\_heavy  
## 243 227 6 pindex enzyme\_test liver\_test gender alc\_mod alc\_heavy  
## 220 228 6 bcs pindex enzyme\_test liver\_test age alc\_mod  
## 222 229 6 bcs pindex enzyme\_test liver\_test gender alc\_mod  
## 225 230 6 bcs pindex enzyme\_test age gender alc\_mod  
## 219 231 6 bcs pindex enzyme\_test liver\_test age gender  
## 244 232 6 pindex enzyme\_test age gender alc\_mod alc\_heavy  
## 237 233 6 bcs enzyme\_test liver\_test gender alc\_mod alc\_heavy  
## 236 234 6 bcs enzyme\_test liver\_test age alc\_mod alc\_heavy  
## 235 235 6 bcs enzyme\_test liver\_test age gender alc\_heavy  
## 246 236 6 enzyme\_test liver\_test age gender alc\_mod alc\_heavy  
## 240 237 6 pindex enzyme\_test liver\_test age gender alc\_mod  
## 231 238 6 bcs pindex liver\_test age alc\_mod alc\_heavy  
## 230 239 6 bcs pindex liver\_test age gender alc\_heavy  
## 245 240 6 pindex liver\_test age gender alc\_mod alc\_heavy  
## 232 241 6 bcs pindex liver\_test gender alc\_mod alc\_heavy  
## 238 242 6 bcs enzyme\_test age gender alc\_mod alc\_heavy  
## 234 243 6 bcs enzyme\_test liver\_test age gender alc\_mod  
## 239 244 6 bcs liver\_test age gender alc\_mod alc\_heavy  
## 229 245 6 bcs pindex liver\_test age gender alc\_mod  
## 233 246 6 bcs pindex age gender alc\_mod alc\_heavy  
## 248 247 7 bcs pindex enzyme\_test liver\_test age gender alc\_heavy  
## 249 248 7 bcs pindex enzyme\_test liver\_test age alc\_mod alc\_heavy  
## 250 249 7 bcs pindex enzyme\_test liver\_test gender alc\_mod alc\_heavy  
## 251 250 7 bcs pindex enzyme\_test age gender alc\_mod alc\_heavy  
## 254 251 7 pindex enzyme\_test liver\_test age gender alc\_mod alc\_heavy  
## 247 252 7 bcs pindex enzyme\_test liver\_test age gender alc\_mod  
## 253 253 7 bcs enzyme\_test liver\_test age gender alc\_mod alc\_heavy  
## 252 254 7 bcs pindex liver\_test age gender alc\_mod alc\_heavy  
## 255 255 8 bcs pindex enzyme\_test liver\_test age gender alc\_mod alc\_heavy  
## R-Square Adj. R-Square Mallow's Cp  
## 4 0.45453888 0.444049241 62.511923  
## 3 0.33434530 0.321544251 87.304176  
## 2 0.17680424 0.160973557 119.800071  
## 8 0.17373244 0.157842683 120.433690  
## 1 0.12009668 0.103175466 131.497104  
## 7 0.03238684 0.013778896 149.588958  
## 6 0.03008638 0.011434194 150.063473  
## 5 0.01420184 -0.004755821 153.339968  
## 30 0.56674092 0.549750369 41.368078  
## 22 0.56159451 0.544402141 42.429624  
## 10 0.52619219 0.507611497 49.732038  
## 16 0.52291945 0.504210410 50.407105  
## 17 0.48866172 0.468609234 57.473427  
## 29 0.48146948 0.461134952 58.956964  
## 26 0.45776685 0.436502801 63.846092  
## 28 0.45527515 0.433913389 64.360053  
## 27 0.45498410 0.433610928 64.420087  
## 11 0.45462075 0.433233332 64.495035  
## 21 0.38256457 0.358351411 79.358019  
## 25 0.35157395 0.326145482 85.750433  
## 23 0.34682661 0.321211965 86.729665  
## 24 0.34310153 0.317340809 87.498034  
## 9 0.27285287 0.244337295 101.988182  
## 15 0.24119435 0.211437270 108.518364  
## 20 0.23204072 0.201924675 110.406477  
## 35 0.21329617 0.182445044 114.272904  
## 19 0.19173531 0.160038658 118.720249  
## 18 0.18665317 0.154757213 119.768540  
## 33 0.17899597 0.146799729 121.347986  
## 36 0.17530704 0.142966134 122.108900  
## 13 0.14533794 0.111821782 128.290606  
## 14 0.14178957 0.108134261 129.022526  
## 12 0.13264651 0.098632646 130.908460  
## 34 0.06526200 0.028605604 144.807821  
## 31 0.04452969 0.007060262 149.084261  
## 32 0.04110299 0.003499185 149.791084  
## 37 0.68412750 0.665175155 19.154821  
## 62 0.67347344 0.653881843 21.352429  
## 76 0.65900001 0.638540013 24.337853  
## 58 0.62874502 0.606469717 30.578532  
## 66 0.62162562 0.598923160 32.047045  
## 43 0.59329539 0.568893111 37.890705  
## 47 0.58930355 0.564661758 38.714100  
## 75 0.58049941 0.555329373 40.530123  
## 51 0.56975132 0.543936398 42.747125  
## 85 0.56937525 0.543537770 42.824696  
## 88 0.56683466 0.540844739 43.348743  
## 87 0.56679389 0.540801526 43.357152  
## 74 0.56270936 0.536471921 44.199666  
## 73 0.56162982 0.535327611 44.422341  
## 61 0.55788037 0.531353187 45.195739  
## 44 0.53654030 0.508732718 49.597541  
## 46 0.53313256 0.505120519 50.300452  
## 59 0.53119788 0.503069755 50.699518  
## 45 0.53046298 0.502290763 50.851105  
## 65 0.52592253 0.497477881 51.787662  
## 60 0.52436176 0.495823467 52.109601  
## 38 0.48963338 0.459011378 59.273003  
## 64 0.48957774 0.458952405 59.284479  
## 63 0.48889410 0.458227745 59.425493  
## 84 0.48356918 0.452583335 60.523860  
## 86 0.48181594 0.450724894 60.885501  
## 50 0.48151865 0.450409771 60.946822  
## 81 0.47248428 0.440833340 62.810336  
## 79 0.46305927 0.430842824 64.754427  
## 82 0.46081087 0.428459525 65.218202  
## 83 0.45580918 0.423157734 66.249898  
## 49 0.45530662 0.422625012 66.353562  
## 48 0.45503310 0.422335081 66.409981  
## 42 0.42658899 0.392184326 72.277129  
## 71 0.40365477 0.367874052 77.007757  
## 69 0.38470475 0.347787033 80.916565  
## 72 0.38258909 0.345544432 81.352961  
## 80 0.36177487 0.323481363 85.646296  
## 78 0.36026762 0.321883672 85.957197  
## 77 0.35574475 0.317089437 86.890125  
## 41 0.31384088 0.272671332 95.533611  
## 40 0.28549447 0.242624142 101.380606  
## 39 0.28171665 0.238619653 102.159855  
## 56 0.27514830 0.231657199 103.514705  
## 70 0.24860492 0.203521216 108.989791  
## 54 0.24663983 0.201438217 109.395129  
## 57 0.24233992 0.196880313 110.282069  
## 68 0.23603433 0.190196391 111.582719  
## 90 0.21853076 0.171642609 115.193168  
## 92 0.21466022 0.167539835 115.991543  
## 67 0.20187460 0.153987077 118.628825  
## 91 0.18127296 0.132149343 122.878311  
## 55 0.16934485 0.119505544 125.338715  
## 52 0.15812764 0.107615301 127.652482  
## 53 0.15011391 0.099120748 129.305469  
## 89 0.07395840 0.018395908 145.014018  
## 97 0.77135652 0.752691750 3.162146  
## 131 0.75014573 0.729749459 7.537284  
## 96 0.70334830 0.679131838 17.190159  
## 94 0.69109391 0.665877082 19.717866  
## 93 0.69104093 0.665819782 19.728793  
## 95 0.68439218 0.658628280 21.100226  
## 136 0.67764578 0.651331145 22.491803  
## 134 0.67559220 0.649109928 22.915394  
## 137 0.67357438 0.646927387 23.331608  
## 111 0.66973376 0.642773249 24.123810  
## 153 0.65967001 0.631887971 26.199653  
## 150 0.65958500 0.631796022 26.217188  
## 152 0.65900072 0.631164048 26.337706  
## 130 0.65869695 0.630835475 26.400366  
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## 128 0.62910663 0.598829618 32.503943  
## 140 0.62381526 0.593106300 33.595390  
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## 109 0.59338586 0.560192871 39.872043  
## 117 0.59168752 0.558355890 40.222358  
## 151 0.58117658 0.546986916 42.390443  
## 149 0.58074361 0.546518597 42.479752  
## 120 0.57314029 0.538294604 44.048083  
## 123 0.56984386 0.534729071 44.728037  
## 122 0.56975265 0.534630420 44.746850  
## 159 0.56939158 0.534239877 44.821327  
## 160 0.56938828 0.534236301 44.822009  
## 161 0.56688775 0.531531652 45.337791  
## 148 0.56272230 0.527026157 46.196997  
## 133 0.56179516 0.526023341 46.388236  
## 135 0.55994900 0.524026474 46.769042  
## 113 0.54131469 0.503870989 50.612731  
## 112 0.54093448 0.503459741 50.691157  
## 115 0.53813760 0.500434548 51.268067  
## 132 0.53273883 0.494595064 52.381667  
## 139 0.52782447 0.489279529 53.395350  
## 141 0.52634731 0.487681789 53.700042  
## 100 0.52616038 0.487479592 53.738601  
## 99 0.49033626 0.448731052 61.128020  
## 138 0.48988217 0.448239898 61.221684  
## 98 0.48977954 0.448128889 61.242854  
## 158 0.48403726 0.441917854 62.427310  
## 119 0.48372197 0.441576828 62.492344  
## 121 0.48190421 0.439610672 62.867294  
## 155 0.47775782 0.435125806 63.722566  
## 157 0.47528866 0.432455086 64.231877  
## 156 0.46706930 0.423564754 65.927280  
## 118 0.45581883 0.411395879 68.247908  
## 106 0.44507907 0.399779399 70.463192  
## 104 0.42895534 0.382339446 73.789024  
## 107 0.42664380 0.379839211 74.265823  
## 145 0.40587583 0.357375900 78.549619  
## 147 0.40368251 0.355003527 79.002035  
## 146 0.38470483 0.334476658 82.916547  
## 154 0.37045802 0.319066834 85.855229  
## 105 0.32795365 0.273092725 94.622578  
## 103 0.31785657 0.262171391 96.705297  
## 102 0.29462320 0.237041419 101.497630  
## 125 0.28055899 0.221829112 104.398645  
## 127 0.27614267 0.217052280 105.309596  
## 144 0.25272363 0.191721479 110.140227  
## 126 0.24840188 0.187046934 111.031672  
## 162 0.22054986 0.156921274 116.776691  
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## 166 0.78090544 0.758083086 3.192498  
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## 164 0.69107329 0.658893420 21.722119  
## 205 0.67980100 0.646446939 24.047246  
## 207 0.67773853 0.644169632 24.472670  
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## 187 0.66985278 0.635462443 26.099260  
## 185 0.66976045 0.635360498 26.118304  
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## 199 0.65874718 0.623200007 28.390006  
## 198 0.63052920 0.592042661 34.210509  
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## 210 0.62407446 0.584915547 35.541925  
## 209 0.62382926 0.584644811 35.592502  
## 178 0.62280841 0.583517619 35.803072  
## 177 0.62271249 0.583411711 35.822857  
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## 184 0.60593384 0.564885287 39.283778  
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## 190 0.60296723 0.561609648 39.895700  
## 192 0.59966959 0.557968503 40.575903  
## 191 0.59812333 0.556261176 40.894848  
## 183 0.59447454 0.552232308 41.647481  
## 213 0.58147589 0.537879631 44.328705  
## 194 0.57315161 0.528688237 46.045749  
## 195 0.57314740 0.528683584 46.046618  
## 196 0.56984521 0.525037423 46.727757  
## 218 0.56940486 0.524551197 46.818589  
## 204 0.56390893 0.518482777 47.952231  
## 189 0.54631855 0.499060065 51.580588  
## 208 0.52837616 0.479248681 55.281552  
## 174 0.52793270 0.478759023 55.373025  
## 176 0.52651554 0.477194241 55.665342  
## 173 0.49054400 0.437475670 63.085168  
## 193 0.48427346 0.430551947 64.378589  
## 217 0.48148917 0.427477622 64.952904  
## 180 0.44751823 0.389968043 71.960068  
## 182 0.44513725 0.387339045 72.451191  
## 181 0.42895897 0.369475534 75.788274  
## 212 0.40587584 0.343987904 80.549618  
## 179 0.33208483 0.262510333 95.770442  
## 197 0.28212933 0.207351139 106.074731  
## 221 0.78141692 0.753512692 5.086996  
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## 241 0.75032059 0.718446626 11.501214  
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## 220 0.71293355 0.676286769 19.213016  
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## 249 0.78148138 0.748228550 7.073698  
## 250 0.78121365 0.747920079 7.128922  
## 251 0.77574320 0.741617169 8.257309  
## 254 0.75032189 0.712327390 13.500948  
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## 253 0.67069882 0.620587766 29.924748  
## 252 0.62558636 0.568610372 39.230066  
## 255 0.78183867 0.743054436 9.000000

plot(all\_models)

## Warning: The `guide` argument in `scale\_\*()` cannot be `FALSE`. This was  
## deprecated in ggplot2 3.3.4.  
  
## Warning: Please use "none" instead.

